Deregulated Wholesale Electricity Prices in Italy

An Empirical Analysis

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Abstract. In this paper we analyze the time series of daily mean prices generated in the Italian electricity market, which started to operate as a Pool in April 2004. The objective is to characterize the high degree of autocorrelation and multiple seasonalities in the electricity prices. We use periodic models with GARCH disturbances and leptokurtic distribution and compare their performance with more classical ARMA-GARCH processes. The within-year seasonal component is built using the low frequencies components of physical quantities, which are very regular throughout the sample. Results reveal that much of the variability of the price series is explained by deterministic multiple seasonalities which interact with each other. Periodic AR-GARCH models seem to perform quite well in mimicking the features of the stochastic part of the price process.

Keywords: Electricity auctions, Periodic Time Series, Conditional Heteroskedasticity, Multiple Seasonalities.

JEL codes: D44, C22, L94, Q40

1. Introduction

Electricity prices as they are now determined in regulated (generally, Pool) markets, where private operators have replaced previously well established public enterprises, present everywhere specific behavioral characteristics. On the one hand, these market-determined prices differ from the prices fixed by governments or public agencies until the end of the last century. In fact, in spite of the very limited storability and transportability of electricity, government-determined prices incorporated little uncertainty in their dynamics as they were generally capped by the imposition of some price ceilings resulting from the implementation of welfare-improving tariff policies. On the contrary, market determined electricity prices are strongly affected by the impossibility of arbitrage between time and space and so they have become very volatile. Yet, time series of current electricity prices differ quite substantially from prices determined in markets for financial assets and other type of commodities since electricity (as well as many physical commodities) cannot be treated like a stock. Electricity prices have specific and somehow unique characteristics (e.g. strong seasonalities and mean reversion) that motivate the use of appropriate time series modelling to study the specific features of their time pattern and to evaluate how prices are affected by temporal demand-supply imbalances, seasonality, transmission congestion and, to a lesser extent, by the features of the mechanism that generates the data (type of auction employed, price rule, degree of market concentration, etc.).

In Italy the privatization of the former public (quasi)monopolist eventually lead to the creation of an electricity Pool which started to operate in April 2004 with some specific legal characteristics such as the presence of Single Buyer on the demand side. In this paper we try and describe the price dynamics of the Italian Pool and compare our findings with those obtained by other authors who analyzed other European markets. We also suggest some methodological estimation procedures that may prove useful for further research in the econometric analysis of electricity prices.

The paper is organized as follows. Section 2 discusses the main characteristics of some European electricity markets for which data availability permits time series data. We try and emphasize differences in the organization and regulation of the markets as well as in the production structure (specifically, electricity generation) that might become important in explaining differences in the econometric results. In Section 3 we describe the main characteristics of the Italian Pool. Section 4 contains a selected review of the existing literature. In Section 5 we introduce and illustrate some general characteristics of the Italian data and take care of the deterministic part of the models. Section 6 describes and motivates the choice of the stochastic models and methods employed to estimate the dynamics of the Italian prices. Results are shown in Section 7 and section 8 concludes.

2. The electricity markets in the European Countries

In this section we describe some general characteristics of the main European electricity exchanges alongside with the main features of each national electricity industry.

The England and Wales (E&W) Electricity Pool started in 1991 after the liberalization of the British electricity market. Since then, competitive electricity markets have been organized in many other countries. Here we consider the Nord Pool, Austria, France, Germany, Netherlands, Spain, and particularly Italy. The key features of most of these experiences have been, in the first place, the privatization and industry reorganization and restructuring of the vertically integrated monopolistic suppliers previously existing. Secondly, the exchange of physical electricity is organized as a competitive wholesale spot market or wholesale auction. Competition has been introduced also at the retail level whereas transmission/distribution, which are still considered natural monopolies, remains under government regulation. All the industry reorganization activity has lead to a separation of potentially competitive elements from natural monopolies.

The wholesale exchange of electricity poses some problems of market architecture and design to regulators. In the first place, it must be decided whether to opt for a centralized Pool or for a decentralized market. In the first case, all the electricity must be allocated through the Pool which is then mandatory; this implies that bilateral contracts are not allowed. All operators, both on the demand and on the supply side, submit hourly or half-hourly bids which are matched by a procedure that minimizes the cost of despatch. A decentralized electricity market like NETA (England) and California, on the contrary, is organized as a series of voluntary forward and spot markets and bilateral contracting is allowed. The advantages of a Pool market over a decentralized one is that demand and supply are continuously matched so that all coordination problems disappear. Advocates of the decentralized market structure emphasize, however, that the Pool may be affected by strategic bidding on the part of those operators having some market power and, as a consequence, the Pool prices do not generally reveal costs. Whilst the issue is still open at the theoretical level, on the empirical side we find many examples, especially in Europe, of non-mandatory electricity Pools, where bilateral contracts are allowed. This choice is probably motivated by the desire to capture the main advantages of the two alternative organization schemes.

Electricity Pools work as multi-unit uniform price auctions: operators submit price/quantity offers which are aggregated by the market operator in order to form a demand (where demand side bidding is allowed) and a supply curve. The equilibrium price and quantity are then determined by the usual crossing condition and all the producers despatched receive the same *System Marginal Price* (SMP) equal to the bid made by the marginal unit called into operation.

As mentioned above, the first European experience of a centralized market for the exchange of physical electricity among producers, distributors and eligible final consumers was the British Pool market which started in 1991 and more recently evolved to a decentralized market (NETA).

The North Pool, who started in 1993, is the oldest electricity market that we consider. The Nord Pool is the unique example of cross-country exchange area and for this reason it might represent a first step towards the integration of the European electricity industry. It links together Norway, which is the founding country, Sweden, who joined in 1996, Finland (1998) and the western part of Denmark (1999). The participation to the North Pool is voluntary.

Omel operated since 1998 as the Spanish market, but it recently became the Iberian Market after the integration of the Portuguese area (2005).

In a similar way, the German EEX (European Electricity Exchange) unifies since 2002 two exchanges previously operating in Leipzig (LPX) and in Frankfurt (EEX).

In the north area of Europe the APX (Amsterdam Power Exchange) operates since 1999; the APX Group is also in charge of the British UKPX since 2003.

Power Next operates since 2001 as the French electricity market. Finally, the Austrian market EXAA is active since 2002.

All the above mentioned electricity markets share some common characteristics, which are summarized in Table 1. First we notice that all the systems are non mandatory markets. Producers and consumers/distributors are allowed to engage in bilateral contracts for the short or long term exchange of electricity. The quantity traded bilaterally is usually included in the total supply recorded in the exchange as zero price offers. A second common characteristic is the existence of demand side bidding. However, the opening of the bidding process to demand has not proceeded at a common pace in all countries. Indeed, following the European Directive 2003/54/EC, all customers have to be considered as eligible by 1/7/2007. This means that at that date all consumers ought to be in a condition to buy electricity directly in the day-ahead market. At present, however, in almost all markets considered, only large (mainly industrial) consumers and distributors are allowed to present the demand bids. Another important common characteristic of the European electricity exchanges is the pricing rule. All day-ahead markets have chosen the system marginal price rule on a hourly basis. This means that 24 auctions are held the day before the delivery, one for each hour of the next day; the last unit despatched, namely the production unit that is necessary to match the last MWh demanded, fixes the closing hourly price for the entire market. Therefore, all units which have been selected by the auction receive the System Marginal Price (*SMP*), for the whole quantity they sell.

The EU Electricity Directive 2003/54 requires each country to implement both legal and functional unbundling for transmission and distribution System Operators. This rule is expected to lead to non-discriminatory network access with tariffs which broadly reflect costs.

Although the provisions of the Directive have usually been transposed into national laws, it is not clear whether network companies have yet modified all aspects of their organization to comply with the new law. The requirement to have legally unbundled and independently managed transmission system operators (TSO) should have been implemented by 1 July 2004. All the countries considered have accomplished to the legal unbundling of network operators, but in some cases there is still overlap between ownership of the TSO and ownership of one (usually the former monopolist) electricity supplier. This is the case, for example, of the French market where RTE is a limited company held by EDF and by the State. In the Nord Pool the State owns both the TSO and some shares in a generating company (see again Table 1).

In all the countries considered the electricity Pool market is organized by a Market Operator (MO, auctioneer). In the former vertically integrated electricity industry, the central operators had full knowledge of operation and fuel cost curves of each unit. They despatched and redispatched the system by using security constrained OPF, which was based on the generation fuel cost optimization. In the deregulated electricity markets the (unconstrained) market dispatch process is similar in the sense that MO collects bids and organizes the despatching of units in a cost minimizing way. This auction-based dispatching does not take transmission conditions into account and so congestions may occur. The main features of the mechanisms implemented to manage congestions has changed in favour of a system compatible with the bid-price-based optimization. When congestion occurs on the transmission line the market operator together with the TSO try and relieve it at the minimum possible cost in a market based way. Either voluntary adjustment bids from generators and loads are used in the optimization procedure to minimize the cost of adjustments, or bids submitted in the day-ahead market are used to change the provisional despatch program when it is unfeasible given the transmission constraints. Market based procedures are totally different from the traditional regulated congestion procedures with centralized mandatory least-cost dispatch. However, the allocation of the congestion costs may be performed in more than one method. One substantial difference depends upon whether or not locational prices are calculated directly in the electricity day-ahead market. Alternatively, separate markets for the congestion management may be implemented when the day-ahead market allocations result unfeasible. In both cases the electricity price results to be different across areas, namely higher in the congested areas and lower in the "exporting" areas, and so prices send the correct signals to operators.

In the old UK Pool, on the contrary, the cost of congestion management was allocated uniformly to all market participants based on system uplifts. However, a uniform price over the network can give incorrect signals for the location of new power plants. Nodal prices are applied in PJM, ISO-NE, ISO-NY whereas zonal prices prevailed in the Nordic Pool and in Italy. Nodal prices would provide the correct location price signals, but they could be very sensitive to operating conditions and network characteristics. Zonal pricing is thought to combine a good performance in sending signals to the market together with a fairly simple implementation. The resolution of bottlenecks is managed by the splitting of markets into zones characterized by different equilibrium prices. In the congested area the price is higher than the one prevailing in the non-congested area. The determination of the different zones is managed differently across markets. In Italy, for example, zones are predetermined on the basis of historical observation and knowledge of the grid line. The day-ahead market IPEX therefore closes with different zonal prices and so it solves the congestion without the need of resorting to a specific congestion market. In the same manner, within Norway - and at the interconnections between the Nordic countries - price mechanisms are used to relieve grid congestion (bottlenecks), by introducing different Elspot area prices[1]. The total geographic market is divided into bidding areas; these may become separate price areas if the contractual flow of power between bid areas exceeds the capacity allocated for Elspot contracts by transmission system operators. When such grid congestion develops, two or more area prices are created. In Norway, because of the topology of its transmission system, most congestion will appear as overloads in certain transmission corridors. In the Nordic Pool, congestion would mostly exist at the same transmission elements since its geographical characteristic leads to power flows in the north-south direction because the large part of hydro plant production is placed in the north while consumption is more spread out in the south. Therefore, the zone definition is easy to implement and the market splitting method is feasible for its congestion management. In Austria, which is an important transit country, congestion on the network occurs because of a high quota of energy that goes through to the lines in order to be delivered abroad. Therefore, the network capacity in this country is extremely valuable and as a result network access tariffs are settled at the highest level with respect to other countries.

The electricity markets considered differ significantly in their underlying productivity structure. This is a very important point since all the issues related to the market design become less severe when the industry is *per se* more competitive. It is well known that electricity can be generated in a variety of ways and using different types of input, which can be either renewable or not-renewable. The cost of the unit of energy supplied depends upon the technology and this influences the shape of the system marginal cost function and hence of the system marginal price. The productive mix of the generating industry is thought to influence the market power of firms, their strategic behavior and finally the prices for energy. For that reason some data about the industry structure must be considered into the analysis. The Nord Pool comprises countries where an high percentage of production comes from hydro resources (56.7%) and still better does Austria where the hydro covers the 69% of total production. Spain and France present similar figures (11.8% and 11% respectively) on hydroelectric production but France has a very high percentage of nuclear production (78%). Netherlands and Germany have a small quota of hydroelectric production (0.1 and 4.2 respectively). Table 2 contains some data which can help us to predict the degree of competitiveness of power markets in the above mentioned countries.

The Nordic area appears to have the more competitive power market. This must be considered together with the high percentage of hydro plants. It is not surprising, then, that Finland, Sweden and Norway prices are well below the EU average, even if they rose slightly due to fuel price increase. The French market is characterized by a high level of concentration (EDF has 90% share of the market) and by a high consumers' protection, which results in low regulated tariffs. Power Next account for a small quota of total energy consumed. This is also the case of Germany where only a small portion of energy is traded on EEX (11%). France and Germany have recently installed new wind plants. All the other markets listed in Table 2 appear to be fairly concentrated and to have a quite low liquidity share.

We therefore conclude that across European countries the level of concentration in generation is still high and this creates the scope for market power and the ability to influence prices. The strong position of incumbent operators has not been eroded in a significant way by investments in generation by new entrants. New generation assets normally entail significant investment costs which are seen as a major barrier to entry. Complex planning procedures and the scarcity of suitable sites have also been named as reasons why the building of new power plants does not take place. Uncertainties associated with the power exchanges have also been considered as entry barriers. Generation is a key issue for competition in the European electricity markets. The generators, due to the characteristics of the electricity market (the non-storability of electricity, the high inelasticity of demand, a very wide spectrum of costs of production and a price equal to the highest offer (SMP) made in power exchanges), are able to influence prices through the use of generation capacity available to them, in particular by either withdrawing capacity (which may force recourse to more expensive sources of supply) or by imposing prices when they are indispensable to meet demand. In the first case, the withdrawal of capacity is profitable if the cost of not producing is more than compensated by the increase in SMP. A large portfolio of low-cost plants facilitate this strategy. In the second case, it is possible to raise SMP even with a relatively small portfolio of plants depending on other offer constraints (e.g. the location of units). The behavior of generators thus can impact significantly on the level of prices, even at a level of lower concentration than in other sectors.

3. The Italian electricity market

The Italian IPEX has been organized on the basis of a Pool system, managed by a market operator ("Gestore del Mercato", GME) who collects the bids, determines the merit order for the dispatching of electricity and is responsible of all the auxiliary services. The Pool initially planned to enter into force by 1^{st} January 2001, in the reality started on the 31^{st} March 2004 as a one-side market. A Single Buyer ("Acquirente Unico"), which has been constituted by the GRTN in 1999, had the responsibility of guaranteeing the supply of electricity to all the captive customers. Demand-side bidding has been introduced since the 1^{st} January 2005.

The Italian electricity market is formed by three different markets, coordinated by the GME: Day-ahead market (MGP), which is the market for the physical exchange of energy, a Rebalancing market (MA) and the Market for the despatchment service (MSD). The three markets operate in a temporal sequence.

Electricity supply is considered a public service in Italy. The opening to competition of production, import, export, purchasing and selling of electricity must be realized in accordance with public service obligations. Several duties and obligations imposed on different operators in the electricity sector fall within the scope of public obligations. In particular, as for the managing of the network, the GRTN had the obligation to ensure the security, continuity and development of the network, to connect to the network all those operators that so request and to ensure priority to the electricity produced on the basis of domestic energy sources. The organizational choice of Italy was initially based on a ISO model, which implied a separation between the ownership and the managing of the network. GRTN managed the line under the guidance of the Ministry of Production Activity, whereas a separate company

(Terna s.p.a) owned at 100% by the former public monopolist (ENEL) had the ownership of the line. On the 1^{st} November 2005 Terna and GRTN merged so that Italy now has the same organizational model of other European countries based with a TSO who both owns and manages the transmission line (see Table 1).

The Single Buyer is obliged to guarantee to captive consumers the security, continuity and efficiency of supply and to apply a unique tariff. A Code of practice for electricity supply has been introduced by the Regulator, regarding customers disconnections for debt, complaints management, meters reading, billing, payments, non-payments handling.

The exchange of electricity in the IPEX, is managed by the GME and scheduled on the basis of the three separated markets mentioned above. In the MGP, where electricity is exchanged for each hour of the following day, producers submit price-quantity bids and the GME organizes the despatch on the basis of the cost minimizing aggregate supply. From January 2005, demand bids, submitted by Single Buyer and by the eligible consumers, are ranked in a decreasing order. The equilibrium between aggregate supply and demand determines the hourly SMP price and the total quantity traded. The SMP is paid to all despatched units. The IPEX is not mandatory so that eligible purchasers and wholesalers may sign bilateral contracts for the exchange of electricity with producers. The provisional program derived from the organized Pool market and from the bilateral transactions is then presented to the TSO who verifies if the electricity flows implied by the program meet the technical constraints of the transmission line. In case of congestion, the market is split into four predetermined zones and new zonal equilibria and prices are calculated.

4. The existing literature

The above discussed modifications in the electricity market organization have stimulated empirical studies of electricity price both in Europe and in the USA. The main elements emerging from these studies are summarized in Table 3.

Bhanot (2000) analyzes electric power prices from twelve Californian regional markets. The objective is to characterize and explain the high degree of autocorrelation and seasonality in power prices and address salient issues that are pertinent for the valuation and hedging of power-based financial contracts. It is shown that price behavior changes with each regional market, so that a firm that seeks to value or hedge power-based contracts must use instruments from the region in which it operates.

Escribano et al. (2002) use average daily prices of several markets and propose a general and flexible model that allows for deterministic seasonality, mean reversion, jumps and conditional etheroskedasticity. They use six nested versions of their model to analyze price behavior in the different markets. Results indicate that AR(1) and GARCH (1,1) with jumps perform better than other versions.

Lucia et al. (2002) present a model which should permit the definition of analytical formulae for derivative pricing. They employ seasonal dummies and sinusoidal functions to deal with seasonality plus an AR(1) autocorrelation structure.

Wilkinson et al. (2002) use Australian data and conduct a non-parametric test of seasonality (peak and off-peak prices) and of log-normality. They obtain mixed evidence: the null hypothesis of equal day effects rejected for some sub-sample periods and not rejected for some other periods.

Carnero et al. (2003) use European data. They argue that the stochastic heteroskedasticity of prices can be correctly modelled when the conditional mean of the time series is properly modelled by means of periodic autoregressive (PAR) processes and proceed to model the seasonalities by means of sinusoids and weekday dummies. PAR(1) models seem to fit best their data. They find evidence of mean reversion in the stochastic part of the model and long memory in the North Pool prices.

Knittel et al. (2005) study the distributional and temporal properties of the price process using several common asset price specification (as well as other less convention models). Results reveal several specific characteristics unique to electricity prices. They use hourly electricity prices (Euro/MWh) of each "zone" if there is a separate market price in each zone. However, with no congestion arbitrage across zones drives the price to a converging level. Then, the degree of divergence is an indicator of no arbitrage opportunities and if a high degree of correlation across "zonal" prices exists, one can use just one zone or the national time series of prices.

Other studies (Fabra and Toro, 2005) investigate collusive vs. cooperative behavior of bidders.

As it is made evident in Table 3, where details on some of the above studies are presented in a comparative way, a common trait of this literature is the adoption of a sort of two-step procedure. A preliminary data analysis is initially conducted in order to gauge from data inspection the main characteristics of the dynamics of the electricity prices. On the basis of this examination it is almost invariably recognized that the models used in the second step for the time series analysis of spot prices have to integrate seasonality and reflect phenomena such as mean reversion, high price-dependent volatility and leptokurtosis. as discussed above, methods to deal with seasonality range from the use of time dummy variables to the application of sinusoids at seasonal frequencies.

In what follows we discuss the results obtained by previous studies by clustering them in sets of specific issues.

a) Seasonality
Real-time balancing and dependency on cyclical demand impose several different seasonal pattern to electricity prices (within day, week, year) almost everywhere. Deidersen and Trück (2002) study price series for Germany, New Zealand and Spain and report strong intra-day pattern and peak during midday. Moreover, monthly mean prices are higher during daytime and weekly seasonal patterns show the presence of weekend effects. Also annual seasonality was found with winter prices always higher than prices recorded in other seasons. Also Knittel et al. (2005), using a model in which the mean was assumed to be time dependent, found that Californian electricity prices show intra-day seasonality and "summer" (rather than winter) effect and Bhanot (2000), using US wholesale transaction prices recorded from 1 January 1995 to 1 June 1998, discovered that the seasonal means for peak and off-peak prices exhibit significant variation across the 12 months and across the delivery points.

b) Volatility
Storage and transmission problems and the need for markets to be balanced in real time are responsible of an unusually high volatility. All the above reported empirical evidence coincide in stressing that there is a strong correlation between the standard deviation and the mean price making the volatility dependent on the price level. Furthermore many time series exhibit some *volatility clustering* making models for conditional etheroskedasticity opportune. When demand approaches and exceeds the limits of the system generation capacity, prices are high and more volatile.

c) Mean reversion
By *mean reversion* we mean the absence of stochastic trends or martingale-like behavior of prices. This is a distinctive feature of electricity prices, with respect to other commodity prices. Electricity prices do not behave as martingales, and the non-deterministic part of the data generating process does not seem to contain unit roots (e.g. no random walk like behavior). When *hourly* prices go

up then they have to move downwards again in a relatively short time. It is thought that they oscillate around some "equilibrium" mean (possibly deterministically time varying). This makes a crucial difference with financial markets. The speed of the reversion is quite informative also in regulatory terms because it displays the time needed by the supply side of the market to react to unanticipated events or the time necessary for the event to be over. The mean reverting nature of electricity prices is generally explained by market fundamentals. Commonly held opinion is that only mean reverting models with jumps allow for brief price spikes (see below) observed in price data and that only the short term mean reversion is the result of seasonal patterns. In the long run electricity prices may revert to some mean. Mean reversion and seasonality are integrated in a model proposed by Lucia et al. (2002) where the price is decomposed into a deterministic and a stochastic component the latter following a Ornstein-Uhlenbeck mean reverting process with zero mean so that price revert to the deterministic function.

d) Spikes and jumps
They are attributed to sudden and strong increase in demand when supply is at the limit of generation capacity or to an unexpected break down of large enough assets. Depending on demand and supply conditions they can also be negative. According to Deidersen and Trück (2002) they are less frequent in market with high level of hydropower generation. Still, spikes are quite pervasive and it is the presence of spikes what makes the forecasting properties of the models used in the literature rather poor. These extreme values can be modelled in discrete time by using stochastic process with leptokurtic marginal distributions or in continuous time by introducing jumps in a Wiener process. Equally important are, at the same time, the problems given by the appropriate modelling of extreme values of electricity prices since price series are highly non-normal with large number of extreme values observations. Byström (2005) models extreme price changes in the Nord Pool and estimates tail quantiles by filtering the return series and then applying an extreme value theory model to the residuals. Like in other studies, the performance of the estimates improves when the model takes into account explicitly the time-of-the-year seasonality of the volatility of the data.

5. Preliminary analysis of the Italian data

In this section we study electricity prices recorded in Italy from April 1st 2004 to January 15th 2006. The data are sampled hourly, but in this study we use daily means[2], leaving the modelling of hourly data for future analysis.

The daily prices are represented in Figure 1 together with the total demand for electricity (daily means as well). The strong weekly season-

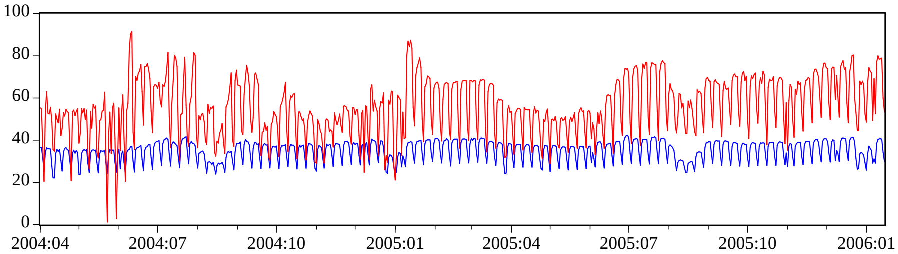

Figure 1. Daily means of hourly prices (above) and of hourly demand (below) for electricity

ality of the prices is clearly due to the seasonality present in electricity consumption. Indeed, the unitary price of electricity changes according to the volume to be produced in a fashion roughly depicted in Figure 2.

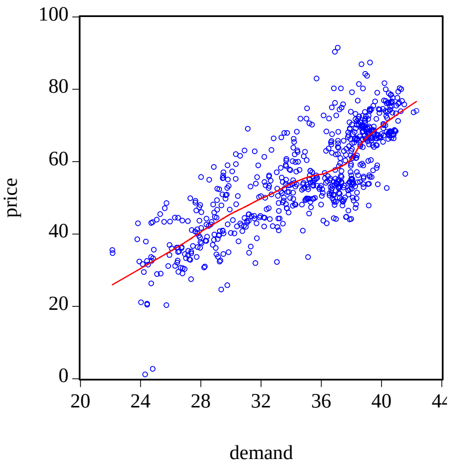

Figure 2. Scatter plot of electricity prices and demand with non parametric curve estimate.

It can be clearly noticed by observing Figures 1 and 3, that in year 2004 the prices have been significantly more volatile compared to the following years. This may be due to a learning phase that the traders have undergone and to the regulation changes that have taken place in January 2005. Furthermore, the first 10-15 days of 2005 witnesses an abrupt increase of the prices not supported by a corresponding rise in the demand. This episode has been followed by an inquiry of the antitrust authority about ENEL (the former public monopolist). The rest of the time series show a greater regularity.

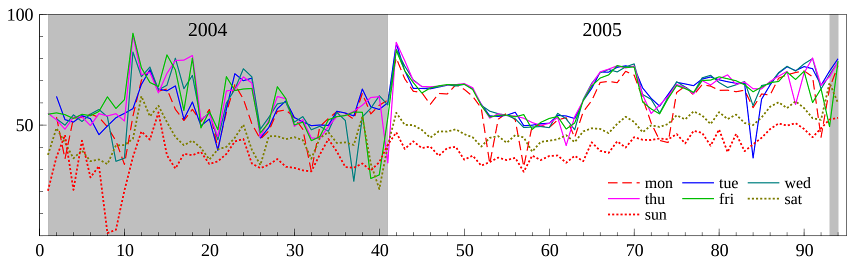

Figure 3. Weekly time series of the seven days.

Table IV reports some descriptive statistics and normality tests and graphs for the weekly time series of each day. It is interesting to notice how the days Tuesday-Friday enjoy a very similar behavior (see Figure 3). By looking at the normality tests (a modified version of the Jarque-Bera statistic is presented in Doornik and Hansen, 1994), one is led to think that the data of Monday-Saturday are normal, but the kernel estimates give evidence of multi-modality for all densities. This is due to the presence of seasonality within the year and a possible trend, which make the data generating process non-stationary and the marginal densities not well-defined. A further problem might be the presence of weekday holidays that make such days behave similarly to Sundays, producing negative skewness.

From the sample autocorrelation function (ACF) reported in Table IV, it is evident the high persistence of linear memory at weekly lags. This leaves three alternatives open: i) the presence of a deterministic weekly seasonality, ii) the presence of 7 seasonal unit roots, iii) the presence of multiple periodic unit roots (Franses and Paap, 2004, ch.4). In previous literature only the first hypothesis has been modelled.

In order to deal with within-the-year seasonality the cited authors have used monthly dummies or sinusoids with frequencies $2\pi/365$ and $4\pi/365$. Since this seasonality is due to the low-frequency components of the electricity demand, and these components tend to be very regular across years, it is very sensible to use them directly instead of approximating them in the mentioned way. By observing the electricity demand series in Figure 1 it easy to notice a higher-than-average consumption in winter and summer with sudden decreases in the two main vacation periods: Christmas holidays (in Italy typically December 24th-January 6th) and August. In order to successfully extract the described features, we have designed a low-pass filter with two different cut-off frequencies: a lower one for "normal" periods and higher one for vacation times. This way, the extracted time series is rather smoothed most of the time, but it does not average out the negative peaks of the two vacation times. The slight trend in the extracted component of the consumption has been eliminated by imposing the same value to December 31st 2004 and December 31st 2005 and adjusting all the other days by linear discounting. Technical details about filtering and detrending are reported in the Appendix. The low-pass filtered series and the final seasonal component are shown in Figure 4.

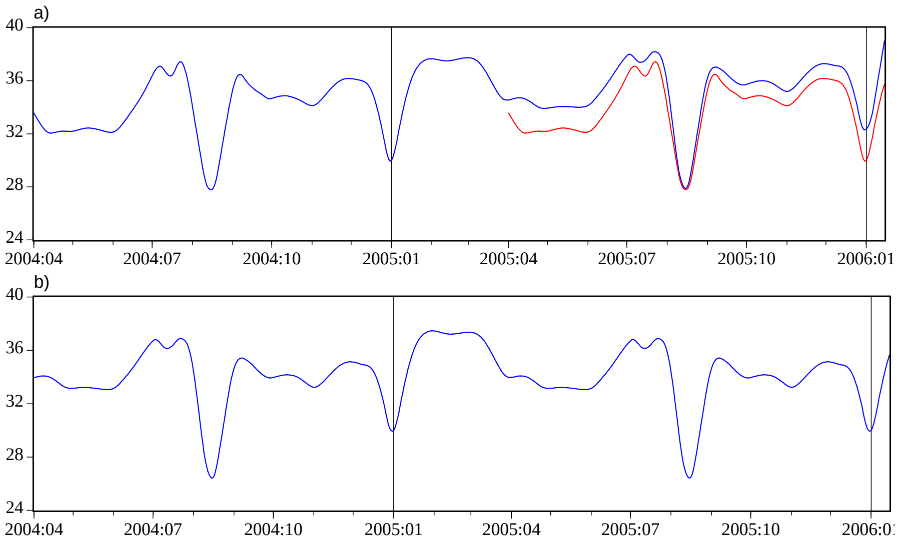

Figure 4. a) band-pass filtered electricity demand (coincident and lagged one year); b) final seasonal component (detrended band-pass filtered electricity demand of year 2005 repeated for all years).

If we assume, at least for the moment, that the price data are generated by the sum of a deterministic component (seasonalities and trend) and a (well behaved) stationary process, the least square estimates of the regression of the prices on the deterministic components are consistent and asymptotically normal (CAN) and the asymptotic covariance matrix of the estimators may be consistently estimated. We estimated the following three nested regressions:

$$y_t \;=\; \tau \cdot t + \sum_{i=1}^{7} \delta_{0,i} \cdot D_{i,t} + \delta_s \cdot S_t + \eta_t, \qquad (1)$$

$$y_t = \tau \cdot t + \sum_{i=1}^{7} (\delta_{0,i} \cdot D_{i,t} + \delta_{1,i} \cdot D_{i,t} S_t) + \eta_t, \tag{2}$$

$$y_t = \tau \cdot t + \sum_{i=1}^{7} \left(\delta_{0,i} \cdot D_{i,t} + \delta_{1,i} \cdot D_{i,t} S_t + \delta_{2,i} \cdot D_{i,t} S_t^2\right) + \eta_t, \tag{3}$$

where $D_{i,t}$ is the daily dummy of day $i = 1, \ldots, 7$ (1 =Mon, 7 =Sun), S_t is the seasonal variable of Figure 4b) and η_t is a stationary process with absolutely summable covariances. The difference among the three regressions is that in equation (1) the within-year seasonality (S_t) enters linearly and cannot influence the within-week seasonality, in equation (2) S_t enters linearly and influences the within-week seasonality and in equation (3) S_t enters quadratically and influences the within-week seasonality. Table V reports summary statistics on the three regression models and on the validity of the constrains imposing the equality of all the parameters relative to the days Tuesday-Friday. The model has been fitted to the whole sample and to the sub-sample February 1st, 2005 through January 15th, 2006. In both samples the constrained model (3) outperforms the others, according to the Schwartz' Bayesian Information Criterion. It is striking how the performance of all the models drastically worsen when the whole sample is considered: for the best fitting model, the standard error of regression is more than double and the R^2 is 20% smaller. These and other considerations have lead us to conclude that omitting the first 10 months will let us produce more accurate models and predictions.

6. Stochastic models for the daily electricity prices

In this section we fit a set of models for the prices, which encompass the regression on deterministic components and take care of the remaining memory that plays an important role in short term forecasting and in derivative pricing.

By looking at the sample ACF and PACF functions it can be noticed the presence of linear memory both in the errors and the squared errors, suggesting the opportunity of ARMA-GARCH models. Maybe curiously, the model in this family that seems to fit the data best is the constrained regression (3) with AR(1,6)-GARCH(1,1) errors:

$$\eta_t = \phi_1 \eta_{t-1} + \phi_6 \eta_{t-6} + \sigma_t z_t \tag{4a}$$

$$\sigma_t^2 = \omega + \alpha \sigma_{t-1}^2 z_{t-1}^2 + \beta \sigma_{t-1}^2. \tag{4b}$$

with z_t i.i.d. standard normal process.

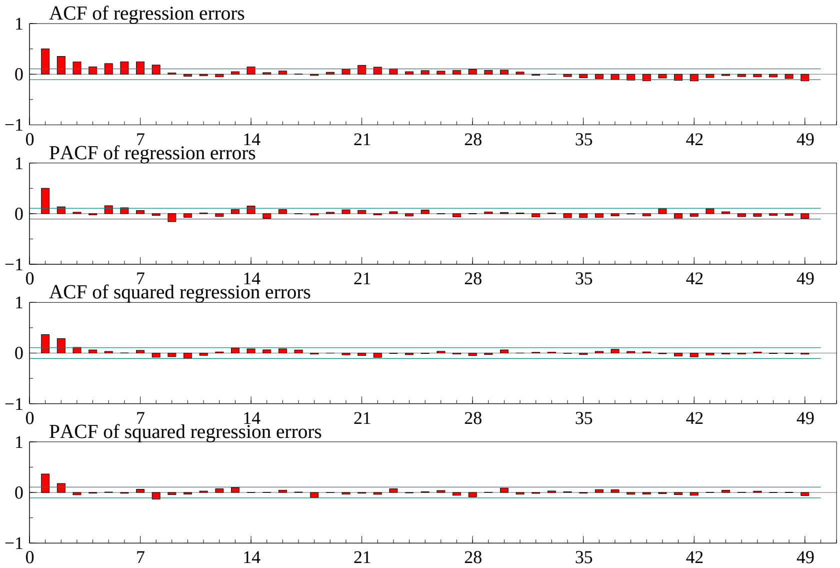

Figure 5. ACF and PACF of regression errors of model (3) with constrains.

Another attractive class of models that could fit the different characteristics of the days better than simple ARMA is that of periodic ARMA-GARCH[3]. A periodic ARMA is an ARMA model with coefficients that vary periodically (with seasons). For example a PARMA(1,1) with period 7 is

$$y_t = \mu_t + \phi_{1,t}(y_{t-1} - \mu_{t-1}) + \sigma_t z_t + \theta_{1,t}\sigma_{t-1}z_{t-1} \tag{5a}$$

with z_t i.i.d. and

$$\mu_{t+7} = \mu_t, \quad \phi_{t+7} = \phi_t, \quad \sigma_{t+7} = \sigma_t, \quad \theta_{t+7} = \theta_t. \tag{5b}$$

A periodic ARMA is a non-stationary model since mean, variance and linear filter depend on time. Nevertheless a PARMA model for daily data has a VARMA representation for the vector of the seven weekly time series. If the VARMA representation has a causal stationary solution, then the process is said periodically stationary. For details on periodic time series models refer to Franses and Paap (2004) and the references therein.

A PARMA model may be enriched with a periodic (but also non periodic) GARCH-type structure by opportunely redefining σ_t in equation (5). From our analyses we expect a non-periodic GARCH-type process to be enough. By looking at the vast GARCH library, we pick the EGARCH of Nelson (1991), since it is easier to adapt to a periodically changing unconditional variance, allows for asymmetry (which implies skewness in the unconditional distribution) and does not impose constrains on the parameters:

$$\log \sigma_t^2 = \log \bar{\sigma}_t^2 + \alpha(|z_{t-1}| - E|z_{t-1}|) + \lambda z_{t-1} + \beta(\log \sigma_{t-1}^2 - \log \bar{\sigma}_{t-1}^2), \quad (6)$$

with $\bar{\sigma}_{t+7}^2 = \bar{\sigma}_t^2$.

In order to identify the orders of a PARMA model the periodic ACF and PACF functions may be used. The seven periodic autocorrelation functions of a periodically stationary process of period 7, x_t, are defined by

$$\gamma_t(k) = E\left(\frac{x_t - \mu_t}{\sigma_t} \cdot \frac{x_{t-k} - \mu_{t-k}}{\sigma_{t-k}}\right)$$

with $\gamma_{t+7}(k) = \gamma_t(k)$. The periodic partial ACF (PACF) are defined in a similar fashion to the non-periodic ones (for definitions and algorithm refer to Sakai, 1982). Figure 6 reports the sample ACF and PACF of the estimated regression (3) errors. It is interesting to observe how the

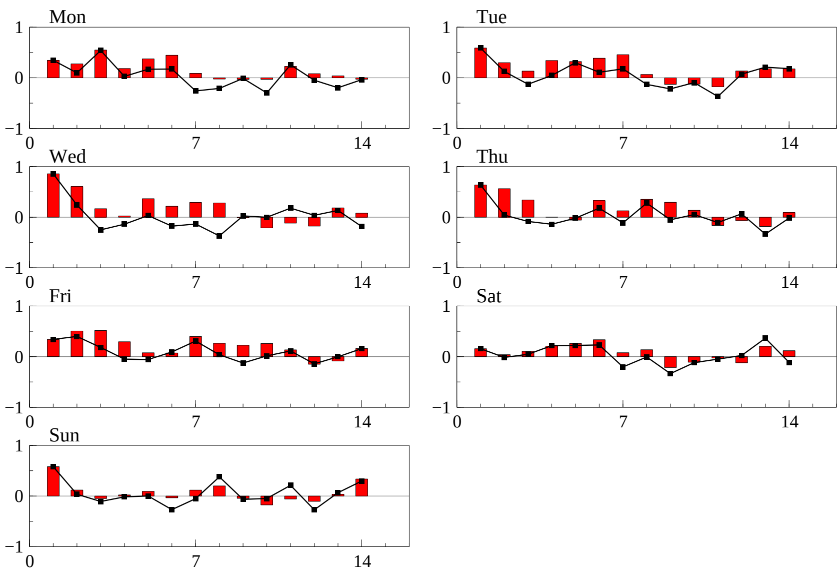

Figure 6. Sample periodic ACF (bar) and PACF (line) of the estimated regression (3) errors.

linear memory changes according to the weekdays. Particularly, it can be noticed the lag 3 partial autocorrelation of Monday, for which the previous Friday carries the most important information, and the scarce influence of previous days on Saturday.

7. Estimation results on Italian data

The process of finding a good model for the Italian data has been incremental: we began with simple models and added complexity gradually, in order to match features of the data that appeared during the modelling process and had not been included in previous models.

M1. Reg-AR(1,6)-GARCH(1,1), $z_t \sim (0,1)$;

M2. Reg-PAR(1), $z_t \sim N(0,1)$;

M3. Reg-PAR(1), $z_t \sim GED^4$;

M4. Reg-PAR(5), $z_t \sim GED$ with

$$\begin{array}{ll}
\text{Mon}: & \eta_t = \phi_{1,1}\eta_{t-1} + \phi_{3,1}\eta_{t-3} + \sigma_1 z_t \\
\text{Tue}: & \eta_t = \phi_{1,2}\eta_{t-1} + \sigma_2 z_t \\
\text{Wed}: & \eta_t = \phi_{1,3}\eta_{t-1} + \sigma_3 z_t \\
\text{Thu}: & \eta_t = \phi_{1,4}\eta_{t-1} + \sigma_4 z_t \\
\text{Fri}: & \eta_t = \phi_{1,5}\eta_{t-1} + \phi_{2,5}\eta_{t-5} + \sigma_5 z_t \\
\text{Sat}: & \eta_t = \phi_{1,6}\eta_{t-1} + \sigma_6 z_t \\
\text{Sun}: & \eta_t = \phi_{1,7}\eta_{t-1} + \sigma_7 z_t;
\end{array}$$

M5. like Mod4. but with EGARCH(1,1) of equation (6).

Table VI reports some goodness-of-fit statistics and diagnostics tests for the five models plus a constrained version of M5. (the insignificant parameters and the insignificantly different parameters have been constrained).

Model M5., specifically in its constrained version, seems to outperform the others, although the simple AR-GARCH works reasonably well, if one is lead by Schwartz' BIC. Table VII reports the constrained estimates. The asymmetry parameter of the EGARCH has been eliminated since not significant.

8. Conclusion

The analysis of Italian electricity prices carried out in this study permitted a good understanding of the most relevant features of these data. The first finding is the significant change of behavior that the data generating process has undergone starting from mid January 2005. This may be due to a learning time needed by the traders involved and by a change of regulation that took place in that period.

Another peculiarity of the Italian prices is the relevant drop during Christmas holidays and summer vacations, that makes a couple of sinusoids or some monthly dummies not fit for modelling with-year seasonality. Thus, an original methodology to deal with this problem has been developed. Furthermore, the interaction of the within-year seasonality with the within-week seasonality has also been modelled.

A slow but significant (increasing) linear trend in the prices has also been noted and modelled. The reasons for this may be found in the relevant growth of the prices of hydrocarbon-based energy sources.

Leptokurtic PAR-GARCH models seem to fit best the different amount of memory of past observations that each weekday carry, as well as the presence of spikes and some form of volatility clustering.

Although the limited length of the price time series leaves some questions open, the models developed in this paper seem to perform quite well. The stability of these models over time must be checked as soon as enough data become available.

Appendix

In order to filter the low frequencies of the daily time series of demanded electricity, we designed a partially model based low pass filter with time varying cut-off frequency. We used the model

$$y_t = \mu_t + \gamma_t^{(1)} + \gamma_t^{(2)} + \gamma_t^{(3)} + \varepsilon_t \tag{7a}$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} \tag{7b}$$

$$\beta_t = \beta_{t-1} + \zeta_t \tag{7c}$$

$$\begin{bmatrix} \gamma_t^{(i)} \\ \tilde{\gamma}_t^{(i)} \end{bmatrix} = \begin{bmatrix} \cos\omega_j & \sin\omega_j \\ -\sin\omega_j & \cos\omega_j \end{bmatrix} \begin{bmatrix} \gamma_{t-1}^{(j)} \\ \tilde{\gamma}_{t-1}^{(j)} \end{bmatrix} + \begin{bmatrix} \kappa_t^{(j)} \\ \tilde{\kappa}_t^{(j)} \end{bmatrix} \tag{7d}$$

with $\omega_j = j \cdot 2\pi/7$, $j = 1, 2, 3$, $\varepsilon_t \sim (0, \sigma_\varepsilon^2)$, $\zeta_t \sim (0, \sigma_\zeta^2)$, $\kappa_t^{(j)}, \tilde{\kappa}_t^{(j)} \sim (0, \sigma_\kappa^2)$. Since the cut-off frequency of the low-pass filter is determined by the signal-to-noise ratio $\rho = \sigma_\zeta^2/\sigma_\varepsilon^2$, we fixed it to 1600 for "normal" days and to 100 for Christmas and Summer vacation time (24Dec-6Jan and July-September). The other unknown variances have been estimated by ML. The filtered series is produced by the Kalman smoother.

The gain of the filter is given by

$$G(\lambda) = \frac{\left[\rho(2 - 2\cos\lambda)^2\right]^{-1}}{1 + \left[\rho(2 - 2\cos\lambda)^2\right]^{-1} + S(\lambda)}$$

where

$$S(\lambda) = \sum_{j=1}^{3} \left[r \left(\frac{4(\cos\lambda - \cos\omega_j)^2}{1 - 2\cos\omega_j \cos\lambda + \cos^2\omega_j} \right)^2 \right]^{-1},$$

ρ is defined as above, r is the signal-to-noise ratio relative to the seasonal component (the estimated value is 48.660.207, meaning that the weekly seasonality is practically time-invariant) and $\omega_j = 2\pi j/7$.

The resulting cutoff frequency for normal times is 0.05π corresponding to a period of circa 40 days. The cutoff frequency for vacation days is 0.10π (ca. 20 days). The two gain functions are depicted in Figure 7.

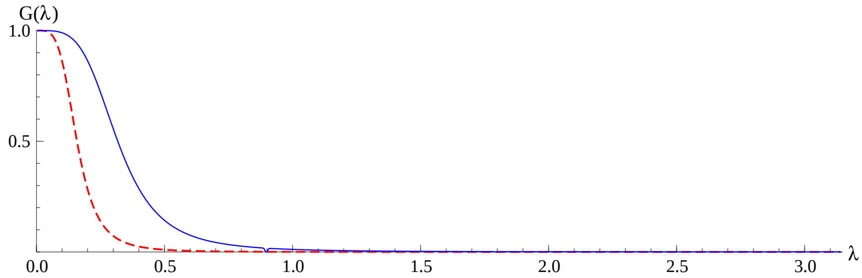

Figure 7. Gains of the filter for normal days (dashed) and vacation days (solid).

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Table I. The European Day-Ahead Electricity Markets

	Nord Pool	**Omel**	**Mibel**	**EEX**	**APX**	**Power Next**	**EXAA**
Country	Norway, Sweden, Finland and Denmark	Spain	Spain and Portugal	Germany	Nederland	France	Austria
Start	1993	1998	2005	1998/2002	1999	2001	2002
Mandatory	No	No		No	No		No
Pricing rule	SMP/h	SMP/h		SMP/h	SMP/h	SMP/h	SMP/h
Demand Side bidding	Yes	Yes		Yes	Yes	Yes	Yes
Grid Owner	Statnett (State-owned) Energinet.dk (State-Owned) Svenska Kraftnät (State Utility) Fingrid (12% owned by the State of Finland)	Red Electrica SA (Public Company)		Four independent companies; EnBW Transportnetze AG E.ON Netz GmbH RWE Transportnetz Strom GmbH Vattenfall Europe Transmission GmbH	TenneT holding BV (State -owned)	RTE (limited Comp. subsidiary of EDF)	Three independent companies: Verbund-APG Tiroler Regelzone AG VKW-Ubertragungsnetz
"Auctioneer" same as Grid Owner?	Yes; Nord Pool Spot AS, is owned by all of the transmission system operators in the Nordic power exchange area and by Nord Pool ASA.	No		No	No	No	No
Who operates?	Producers, distributors, industrial companies, energy companies, trading representatives, large consumers and TSOs	Producers and self-producers, distributors, resellers qualified consumers.		Production/distribution companies, large consumers, industrial end-users, brokers and traders. All of these can be active as buyer or supplier	Production/distribution companies, large consumers, industrial end-users, brokers and traders. All of these can be active as buyer or supplier	Producers, suppliers, Industrial consumers, financial institutions traders	Production/distribution companies, large consumers, industrial end-users, brokers and traders. All of these can be active as buyer or supplier
Financial market?	Yes	No		Yes		Yes (from 25/11/2005)	Yes
Management of bottlenecks	Internal after market splitting	After market congestion bids		Market based (bidding zones)	Market based	Market coupling (Belpex)	Market based (three control zones)
% of hydro	56.7	11.8		4.2	0.1	11	69

Table II. Wholesale Market Positions - end 2004

Country	Number of companies with at least 5% share of production capacity	Total share of the 3 largest producers	Liquidity
Austria	5	54%	3%
Finland	10	40%	42%
Sweden	10	40%	42%
Norway	10	40%	42%
France	1	96%	3%
Germany	5	72%	11%
Italy	5	65%	21%
Netherland	4	69%	12%
Portugal	3	76%	-
Spain	3	69%	92%

Table III. Some previuos analyses.

	Atkins et al. ('02)	Knittel et al. ('05)	Carnero et al. ('03)	Escribano et al. ('02)	Lucia et al. ('02)
Hourly electricity prices generated by	Alberta Power Pool	California 26 "zones"	APX, NordPool, EEX and Powernext	Average *daily* prices of Nord Pool, Argentina, Victoria (Australia), New Zealand, Spain	Nord Pool
Period analysed	01/01/98 to 30/09/2001	01/04/1998 to 30/08/2000	01/01/01 to 08/06/03 04/01/93 to 14/11/99 01/10/01 to 08/06/03 03/12/01 to 08/06/03	01/01/93 to 30/11/99 01/01/95 to 30/09/00 01/07/94 to 12/12/99 01/10/96 to 31/09/00 01/10/98 to 31/12/00	01/01/93 to 31/12/99
Raw data reveal:					
Seasonality	No specific test	a) Regular intra-day pattern b) Weekday/Weekend cycle c) Seasons	Strong weekly patterns of average daily prices in all markets	Only weekly and monthly considered	Strong seasonal pattern along the year with the exception of 1996
Volatility	High level of persistent and cluster volatility	Time varying and volatility clustering	Volatility clustering	Volatility periodicity considered with respect to the 4 seasons of the year	Strong volatility with seasonal differences
Mean reversion		Yes	Yes	Yes (slow for Nord Pool)	
Spikes and jumps		Jumps from every 20 to 33 hours (estimated). Extreme values are present Use jumps diffusion models and exponential GARCH	Yes	Important price spikes and jumps detected GARCH process with jumps	Yes with Extreme values
Time series Model					
Method	Conditional ML	Conditional ML (Various versions starting from a Ornstein-Uhlenbeck process)	Various regressions ARIMA structure and PAR models	ML on 6 nested models GARCH-Poisson-Gaussian outperforms constant volatility and pure jump models (except Spain)	Non linear LS (Various versions starting from a Ornstein-Uhlenbeck process)
Long memory	yes		Nord Pool shows specific features		yes

Table IV. Descriptive statistics, normality tests and kernel density estimates (top graph) for each day and sample ACF of the whole time series (bottom graph).

	mon	tue	wed	thu	fri	sat	sun
Means	58.04	61.96	62.24	62.84	61.94	46.66	37.93
St.Dev.	10.92	10.06	11.42	11.08	11.17	7.83	8.75
Skew	-0.487	-0.113	-0.488	-0.054	-0.363	-0.073	-1.334
Kurt	2.977	2.449	3.220	2.595	3.804	3.643	7.453
Min	29.06	35.16	24.68	32.89	25.87	21.14	1.21
Max	80.26	86.91	84.25	90.38	91.49	69.09	55.75
Norm (sig)	0.103	0.590	0.141	0.881	0.099	0.166	0.000

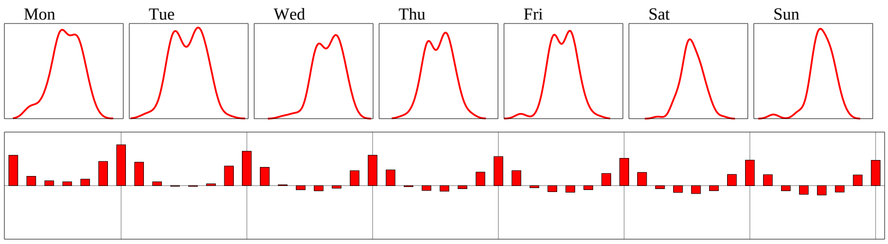

Table V. Diagnostics for the regression models of equations (1)-(3).

Whole sample	eq. (1)	eq. (2)	constr	eq. (3)	constr
R^2	0.67	0.68	0.68	0.70	0.69
S.E. of Regression	7.93	7.85	7.83	7.68	7.65
LogLik	-2281	-2272	-2273	-2254	-2255
BIC	7.06	7.09	7.03	7.10	*7.02*
Wald Test Sig*		0.52		0.55	
Feb2005-Jan2006	**eq. (1)**	**eq. (2)**	**constr**	**eq. (3)**	**constr**
R^2	0.84	0.87	0.87	0.92	0.92
S.E. of Regression	4.97	4.52	4.51	3.49	3.48
LogLik	-1050	-1014	-1016	-920	-924
BIC	6.17	6.06	5.98	5.64	*5.51*
Wald Test Sig*		0.32		0.23	

*Wald test for the equality of all the parameters relative to Tuesday-Friday.

Table VI. Goodness-of-fit statistics of the various models.

	M1.	M2.	M3.	M4.	M5.	M5.c
LogLik	-834	-840	-813	-804	-792	-798
N. of Coefs.	15	27	28	30	33	23
AIC	4.88	4.97	4.82	4.78	4.73	4.71
BIC	5.08	5.27	5.13	5.11	5.09	4.96
$Q(10)$ Sig.	0.229	0.007	0.000	0.000	0.005	0.005
$Q(10)^2$ Sig.	0.855	0.012	0.221	0.724	0.989	0.989

$Q(10)$ is the lag 10 Box-Lijung statistics on the standardized residulas.
$Q(10)^2$ is the lag 10 Box-Lijung statistics on the squared standardized residuals.

Table VII. Estimates of model 5. constrained (only the parameters of the stochastic part are reported).

	Coefficient	Std.Error	t-Ratio	Prob.
$\phi_{1,1}$	0.251	0.098	2.555	0.011
$\phi_{1,2} = \phi_{1,4} = \phi_{1,5}$	0.776	0.047	16.504	0.000
$\phi_{1,3}$	0.985	0.073	13.475	0.000
$\phi_{1,7}$	0.827	0.103	8.034	0.000
$\phi_{2,5}$	0.211	0.075	2.823	0.005
$\phi_{3,1}$	0.388	0.073	5.288	0.000
α	0.361	0.101	3.561	0.000
β	0.695	0.150	4.621	0.000
$\sigma_1 = \ldots = \sigma_7$	1.349	0.368	3.662	0.000
GED's r	0.925	0.082	11.296	0.000