Procurement Auctions in the Presence of a Mixed Oligopoly

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Abstract
Should a welfare maximizing public firm participate to a procurement auction alongside private firms? In a standard private value model with distributionally symmetric bidders, we show that a regulator may reduce his expected procurement cost if he can admit the public firm to a specific procedure consisting of a two-stage mechanism, a negotiation followed by an auction. We show that the probability that the public firm is awarded the contract is higher if the regulator resorts to the two stage mechanism. When the two-stage mechanism is legally forbidden or not implementable, we show that expected procurement costs depend upon the auction procedure selected. First price sealed bid and English auctions perform better than the second price sealed bid auction. Moreover, we show that the policy of revealing the presence of a welfare maximizing public firm to private firms does not generate further cost saving since an English auction generates the same expected procurement cost as the first price sealed bid auction.

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1 Introduction and content of the paper
Economic literature on mixed oligopolies claims that one public firm can be used as a policy instrument to regulate concentrated markets from “inside”. De Fraja and Delbono (1989), Cremer et al. (1989) have considered the interactions between one public and n private firms producing one homogeneous good and competing by means of market instruments only. Private firms are assumed to maximize profits whereas the public firm maximizes social welfare. In the Nash equilibrium of the full information game, De Fraja and Delbono (1989) show that social welfare may be higher when the public firm mimics the private firms’ behavior than when the public firm behaves in the welfarist way. This result implies that in some circumstances the value of a public firm in enhancing the social welfare may be the same of an additional private firm entering into the
market. Therefore, from the Government point of view, a public firm might have the same role of a private firm.

When profit maximizing agents compete at auction, Bulow and Klemperer (1996) show that nothing is worth more for an auctioneer than an additional bona fide bidder.

In this paper we try and put together the above two streams of literature and, in particular, we explore the case in which the additional bidder is a public firm who competes against \( n \) private firms in an auction. In our model, it will be assumed that the Government faces a selection problem for the awarding of a procurement contract whose obligation could be equivalently fulfilled by any one of \( n + 1 \) firms. The \( n \) private firms maximize profits whereas the one publicly owned is assumed to maximize welfare.

We show that the expected payment for the Government might depend upon: (i) whether or not he ought to treat equally public and private firms and - in the former case- (ii) upon the auction format selected.

We show that the lowest procurement payment is obtained when the Government can design a mechanism that strictly favors the public firm. In this connection, we show that a welfare maximizing public firm should never participate to an auction side-by-side to the private firms. We also show that the public firm is strictly better off when excluded from the auction. We consider a two step procedure in which the Government first has a deal with the public firm and then conducts an auction among the \( n \) private firms. The above procedure allows the Government to save on procurement cost but it may generate an ex-post inefficient allocation of the contract.

On the contrary, when the Government treats equally all firms, we show that the expected procurement payment depends upon the auction format: English and first price sealed bid auctions perform better than the Vickrey (1961) auction. In particular, we show that the expected procurement payment resulting from a second price sealed bid auction is equal to the one prevailing in the case in which the public firm behaved as a private one. Therefore the public firm and private firms are worth the same at a second price auctions. This is not the case in an English and first price sealed bid auction: the expected procurement cost is strictly less when the public firm acts as welfare maximizer.

The paper is organized as follows. In the next section we will introduce the model, whereas in section 3 we analyze the case in which the public firm goes to the auction. In section 4 we consider the case in which the regulator can exclude the public firm from the auction. Section 5 contains some concluding comments.

2 The model

Suppose that the Government wants to minimize the expected payment of procuring a predetermined quantity, \( \bar{y} \), of some private good or service (e.g. weapons, food, etc.). The potential sellers of the good operate in a mixed oligopoly so that the buyer faces a set \( F = (0; 1, ..., n) \) comprising \( n + 1 \) risk
neutral firms. We assume that \( n \) firms belonging to \( F \) are private firms whereas firm 0 is publicly owned. The cost of supplying the output \( \bar{y} \) is a linearly increasing function of a private characteristic \( \varphi \), \( C(\bar{y}, \varphi) \). The private characteristic may be interpreted in many ways. It can be considered either as a pure inefficiency cost term or as a measure of the opportunity cost of resources, or as some minimum profit requirement. Let the \( n \) private firms be indexed by \( i \) whereas the public firm be indexed by 0. For each firm \( i \), we call \( t_i \) the minimum payment required for supplying \( \bar{y} \):

\[
t_i = C\left(\bar{y}, \varphi_i\right) = t(\varphi_i)
\]

(1) is the "internal procurement cost" for firm \( i \), whereas for firm 0 we have:

\[
t_0 = C\left(\bar{y}, \varphi_0\right) = t(\varphi_0)
\]

We assume first that the Government wants to assign the procurement contract by an auction under asymmetric information about the firms’ internal characteristics \( t(\varphi) \). As in standard private value auction models\(^1\), bidders have ex-ante symmetric costs, which are assumed to be independent. Each firm observes her own cost only \( t_j \), \( \forall j \in F \), and the uncertainty about the opponents’ costs is modeled by means of a common distribution \( G(\cdot) \) with a positive density \( g(\cdot) \), defined over a support \([a, d]\), which is common knowledge among all bidders and the Government.

A source of asymmetry in our model is given by different objective functions of bidders: private firms are profit maximizing whereas the public firm is welfare maximizing. If the public firm has the objective of maximizing welfare and the terms of the contract are known, then both Government and public firm will try to minimize the procurement cost. This fact is common knowledge among the buyer and the sellers but, depending on the auction format selected, the sellers may be unaware that one of the opponents is a public firm.

The presence of a public firm among potential sellers gives to the Government a "regulatory instrument" which can be used inside or outside the auction to put competitive pressure on private firms.

In this connection, it might be interesting to evaluate what is the best procurement policy for the Government. The Government’s problem can be framed in terms of deciding: (i) should the public firm receive an equal treatment with respect to private firms and (ii) which kind of auction procedure must be implemented to allocate the contract.

3 The public firm goes to the auction

We first analyze the case in which the Government is restricted to treat equally all potential suppliers\(^2\). An auction with \( n + 1 \) bidders should be organized and

\(^1\)All the assumptions that have been made so far characterise a standard private value auction model, as in Riley and Samuelson (1981).

\(^2\)This for example can be required by a law or a treaty directly enforceable.
the problem for the Government becomes essentially the choice of the auction format. The Government can decide to allocate the contract by means of an open English auction or he can choose a sealed bid auction, namely a first price (FPSB) or a second price (SPSB) procedure. We assume that the different auction mechanisms which can be used to allocate the contract, namely open or sealed bid, convey a different amount of information to participants: in the English auction all the n private bidders know that the(n + 1) participant is a welfare maximizing public firm. This can happen because, for example, bidders compete into the same room. On the contrary, we assume that in sealed bid procedures bidders know only the total number of opponents and they are not informed that one of them is a public firm.

We first consider the SPSB auction introduced by Vickrey (1961). Private (profit maximizing) firms participating to a SPSB auction have a (weakly) dominant strategy of asking their true cost of the procurement. The expected profit accruing to the winner is equal to the expected difference between the amount bid and the price actually received, i.e. the second-lowest bid. The result is Pareto optimal since the most efficient firm always obtains the contract.

On the other hand, a welfare maximizing public firm cannot do better than bidding her true cost\(^3\). In so doing, she will earn some “unwanted” profit in the case of winning, as it happens to private firms.

As a consequence, the expected procurement cost from a SPSB auction in which all the n + 1 bidders -public and private- participate is equal to the expected procurement cost that the Government would obtain from an auction with n + 1 private firms. The latter is given by the expected value of the second-lowest order statistics in the sample of costs:

\[
E[P]_{SP}(n+1) = (n+1) \int_a^d t_1(1 - G(t_1))^n + \int_{t_1}^d (1 - G(t_2))^n dt_2 \int g(t_1) dt_1 \quad (2)
\]

If, for example, the parent G(t) is uniform on the unit interval, (2) can be solved as:

\[
E[P]_{SP}(n+1) = \frac{2}{(n+2)} \quad (3)
\]

As far as a SPSB auction is considered, a public firm “is worth” the same for the Government as a private firm. Moreover, we notice that both the bidding strategy of private firms and the expected procurement cost for the Government do not change if bidders were informed that one of the opponents is a welfare maximizing public firm. As a consequence, in a SPSB procedure the policy of revealing the identity of one of the bidders would not affect the result.

Consider now the case of one public and n private firms competing for the procurement in a first price sealed bid (FPSB) auction. Since the public firm operates in "the social interest", she will bid at cost\(^4\). On the other hand, the n

\(^3\)We assume that side payments are not allowed here.

\(^4\)Bidding at cost the public firm also satisfy a balanced budget constraint requirement.
private firms will adjust their bidding behavior taking into account that there are \((n+1)\) bidders. The optimal bid for a private firm \(i\) is therefore given by:

\[
b(t_i) = t_i + \frac{\int_{t_i}^{d} [1 - G(s)]^n \, ds}{[1 - G(t_i)]^n} \tag{4}
\]

When \(G(.)\) is uniform on the unit interval, the optimal bid is:

\[
b(t_i) = \left[ t_i \left( \frac{n}{n+1} \right) + \frac{1}{n+1} \right]
\]

and bids are uniformly distributed in the interval \(\left(\frac{1}{n+1}, 1\right)\).

Now let us evaluate the expected procurement payment for the Government when the public firm goes to the FPSB auction. If the public firm wins the FPSB auction, the Government will pay \(t_0\) for the procurement. On the contrary, the Government will pay the smaller bid out of \(n\) private firms’ bids, if this is lower than \(t_0\). The expected procurement cost evaluated in ex-ante terms, namely before all sealed envelopes are opened, is given by:

\[
E[P_{fp}^{(n)}] + PF = E_{t_0} \left[ (1 - G(t_0))^n \right] +
\]

\[
+ E_{t_0} \left( \int_{a}^{t_0} \left( \tilde{t}(1) + \frac{\int_{\tilde{t}(1)}^{d} [1 - G(s)]^n \, ds}{[1 - G(\tilde{t}(1))]^n} \right) g_{(1),n}(\tilde{t}_1) d\tilde{t}(1,n) \right)
\]

where

\[
g_{(1),n}(t_1) = ng(t_1)(1 - G(t_1))^{n-1}
\]

indicates the density of the lowest order statistics in the sample of size \(n\).

Equation (5) can be rewritten as:

\[
E[P_{sp}^{(n)}] + PF = (n + 1) \int_{a}^{d} \left( 1 - G(\tilde{t}) \right)^n g(\tilde{t}) \, d\tilde{t} +
\]

\[
+ n \int_{a}^{d} \left( \int_{a}^{d} (1 - G(s))^n \, ds \right) g(\tilde{t}_0) \, d\tilde{t}_0
\]

The first result of the paper can be summarized into the following Proposition 1.

**Proposition 1** When a welfare maximizing public firm is comprised into the set of bidders, a first price sealed bid auction generates a lower procurement payment for the Government than a second price sealed bid procedure:

\[
E[P_{sp}^{(n+1)}] > E[P_{sp}^{(n)}] + PF
\]
Proof. Using (2) and (6):

\[
(n + 1) \int_a^d \left[ \tilde{t} \left( 1 - G (\tilde{t}) \right)^n + \int_{\tilde{t}}^d (1 - G (\tilde{t}))^n \, d\tilde{t} \right] g (\tilde{t}) \, d\tilde{t} >
\]

\[
(n + 1) \int_a^d \tilde{t} \left( 1 - G (\tilde{t}) \right)^n g (\tilde{t}) \, d\tilde{t} + n \int_a^d \left( \int_{\tilde{t}}^d (1 - G (s))^n \, ds \right) g (\tilde{t}) \, d\tilde{t}
\]

The first term on the lhs cancels out with the first term on the rhs of the inequality. Simplifying further we obtain that the difference between the expected payments is:

\[
\int_a^d (1 - G (\tilde{t}))^n G (\tilde{t}) \, d\tilde{t} > 0 \quad (7)
\]

If we take the uniform distribution on \([0, 1]\) as example, the expected savings for the Government can be written as: \([(n + 1) (n + 2)]^{-1}\). The two procedures become equivalent as \(n\) grows, as it can be seen from Figure 1, but when the number of private firms is small, expected cost savings may be large.

Figure 1: Expected procurement cost in the FPSB and in the SPSB auctions when the public firm participates.

The above result implies that the revenue equivalence between first and second price auction procedures fails when a welfare maximizing public firm belongs to the set of bidders, unless the number \(n\) of private firms competing at auction becomes quite large. In this connection, a FPSB auction procedure must be strictly preferred to a SPSB auction. The public firm bidding at cost into the FPSB auction, is worth more for the Government than one additional private firm that would bid according to the equilibrium bidding strategy \(b (t)\).
We next consider an open auction format, in which the presence of a welfare maximizing public firm is common knowledge among private bidders. In the English auction bidders submit oral bids to the auctioneer and each private firm \(i, i \in (1, ..., n)\) stays active until the current price reaches his private cost \(t_i\), then he leaves the auction. We assume the the public firm will always bid at \(t_0\), even in the case in which she remains in competition with the last private firm\(^5\).

Therefore if the English auction format is selected to allocate the contract, the Government will pay \(t_0\) when there is not a private firm bidding less than \(t_0\). The payment will be equal to the second lowest of firms’ costs, ranging from \(t_0\) to the upper bound of the distribution when at least the lowest of private firms’ cost is lower than \(t_0\). Taking the expectation over \(t_0\), the expected procurement cost from the English procedure is given by:

\[
E[P]_{t_0}^{E,F} = E_t \{[t_0 (1 - G(t_0))^n] + n \int_{t_0}^{t_1} t_1 (1 - G(t_1))^{n-1} dt_1 + n \int_{t_0}^{t_2} (1 - G(t_2))^{n-1} dt_2 + -n \int_{t_0}^{t_1} t_1 (1 - G(t_1))^n dt_1 \}
\]

A simple manipulation shows that (8) is equal to (6). If costs are uniformly distributed on the unit interval, both (8) and (6) generate:

\[
E[P]_{t_0}^{E,F} = \frac{2n + 1}{(n + 2)(n + 1)}
\]

Therefore we conclude that the English auction is equivalent to the FPSB auction from the Government point of view\(^6\). The policy of revealing the presence of the public firm does not improve upon the results obtained through a sealed bid procedure with a first price rule.

4 The public firm stays outside the auction

In this section we consider a different competitive scenario in which the Government discriminates between the public firm and private firms. Auction theory indicates that if the Government knew with certainty before the auction the true private cost \(t_0\) of the public firm, he could use it to fix a ceiling level for

\(^5\)The latter assumption on the public firm’s behaviour is introduced in order to emphasise her welfare maximising objective. Another possibility is to assume that when the public firm reaches the final round and compete with the last remaining private firm, she would diminish of a given amount the last bid made by the opponent. This new assumption would slightly change the result on expected procurement cost, as we will describe later.

\(^6\)This result would be modified if we assumed that the public firm will not be willing to bid at cost in the final auction round. In that case, it is easy to show that the expected procurement cost would be lower in the FPSB procedure.
the payment in the auction to be held among the private firms only\textsuperscript{7}. Let us
analyze the following scheme.

1. At time $T = 1$, the Government asks the public firm to announce the
minimum payment at which she would be willing to fulfill the contract.
Suppose she declares a value $\hat{t}$.

2. At time $T = 2$, the Government organizes an auction where only $n$ private firms participate. In that auction the Government fixes a ceiling value $t^* = t(\hat{t})$.

3. After opening the envelopes, the procurement is awarded to the public firm if there is no a private firm bidding less than $t^*$ during the auction. The public firm receives $\hat{t}$. Alternatively, the procurement is awarded to one private bidder that bids less than the ceiling value $t^*$. The price paid by the Government is to be determined on the basis of the auction procedure implemented\textsuperscript{8}, namely first or second price.

In this scenario, we show that the public firm finds optimal to reveal her true cost $t_0$ and, in doing so, she is strictly better off when she stays outside the auction rather than when she participates to it. This happens because the public firm anticipates the optimally set ceiling value $t^*(\hat{t})$ in the auction that is held in the second stage. The second stage auction is a standard auction played by $n$ profit maximizing firms and the expected procurement cost does not depend upon the pricing rule selected by the Government.

We now consider the “interim” expected payment of the Government, at the beginning of stage 2 of the game. The expected payment is given by two terms: if no private firm asks a payment less than $t^*$ in the auction, the contract goes to the public firm with an expected payment of $\hat{t}[1 - G(t^*(\hat{t}))]n$, $\forall \hat{t} \in [a, d]$. If, on the contrary, the contract is awarded to some private firm that participates to the second stage auction and bids below $t^*$, the expected payment is given by:

$$E[P] = n \int_a^{t^*} \left[ t_1(1 - G(t_1))^{n-1} + \int_{t_1}^{t^*} (1 - G(t_2))^{n-1} dt_2 \right] g(t_1) dt_1 \tag{9}$$

All together, the “interim” expected payment for the Government at the beginning of stage two, $E[P]_{TS}$, when $\hat{t}$ is known, is given by:

\textsuperscript{7}In general, even a private firm could be excluded from the auction but she would reveal her true cost only when this policy would guarantee a level of profits at least equal to the ones accruing to her into the auction. Bulow and Klemperer (1996) show that in this hypothesis nothing is worth more for the auctioneer than a “bona fide” auction participant.

\textsuperscript{8}In the case of a first price auction the winner receives an amount equal to his bid, whereas in a second price auction, he receives an amount which can be equal to the second lowest bid, when the latter is lower than the ceiling, or equal to the ceiling itself.
\[ E[P]_{TS} = \hat{t}[1 - G(\hat{t})]^n + n \int_a^{\hat{t}} [1 - G(t)]^{n-1} \left[ t_1 + \frac{G(t)}{g(t_1)} \right] g(t_1) dt_1 \quad (10) \]

Minimizing (10) with respect to \( t_* \), we obtain a standard result of auction theory:\footnote{See for example, Laffont and Maskin (1980).}

\[ t_* = \hat{t} - \frac{G(t_*)}{g(t_*)} \quad (11) \]

If bids are uniformly distributed on \([0, 1]\), \( t_* \) is given by \( t_* = \frac{1}{2} \hat{t} \).

Therefore, once \( \hat{t} \) is known, the Government finds optimal to set a ceiling in the auction lower than the amount \( \hat{t} \) asked by the public firm during the first stage. In so doing, the Government accepts the probability of awarding the contract to the public firm even if there exists a private one that can perform the contract at a lower cost; it happens when \( t_* < t(1) < t_0 \). Given this result, we can now state the following.

**Proposition 2** The policy of excluding the welfare maximizing public firm from the auction leads to a differential treatment in favor of the public firm itself. The public firm finds optimal to reveal her true cost during the private deal and she strictly prefers to be excluded from the auction.

**Proof.** The public firm strictly prefers to be excluded from the auction, since she gets the contract with higher probability. This is true because for any value \( \hat{t} \) announced in the first stage:

\[ [1 - G(\hat{t})]^n < [1 - G(t_*))]^n. \]

Moreover, since \( t_* \) is a decreasing function of \( \hat{t} \), the public firm maximizes the probability of getting the contract when \( \hat{t} = t_0 \). \( \blacksquare \)

The expected procurement cost for the Government at the beginning of stage 1, namely, before \( t_0 \) is known, is given by:

\[ E[P]_{TS} = \int_a^{\hat{t}_0} [1 - G(t_*)]^n g(\hat{t}_0) d\hat{t}_0 + \int_a^{\hat{t}_0} \int_a^{\hat{t}_*} \left( \hat{t}_0 (1 - G(\hat{t}_0))^n \right) g(\hat{t}_0) d\hat{t}_0 + \int_a^{\hat{t}_0} \int_a^{\hat{t}_*} (1 - G(\hat{t}_0))^{n-1} d\hat{t}_0 \]

\[ + \int_a^{\hat{t}_0} \int_a^{\hat{t}_*} (1 - G(\hat{t}))^{n-1} d\hat{t}_0 - n \int_a^{\hat{t}_0} \int_a^{\hat{t}_*} (1 - G(\hat{t}))^n d\hat{t}_0 \]

\[ + n \int_a^{\hat{t}_0} \int_a^{\hat{t}_*} (1 - G(\hat{t}))^{n-1} d\hat{t}_0 \]

\[ = t_0^n - G(t_0) + n \int_a^{\hat{t}_0} (1 - G(\hat{t}))^{n-1} d\hat{t}_0 - n \int_a^{\hat{t}_0} (1 - G(\hat{t}))^n d\hat{t}_0 \]

9 See for example, Laffont and Maskin (1980).
If we again consider the uniform distribution on the unit interval, the above rewrites as follows:

\[ E[P_{TS}] = \frac{2^{-n} + 2n}{(n + 2)(n + 1)} \]

We now compare the expected procurement cost for the Government under the two alternative scenarios, namely a two stage procedure with a differential treatment in favor of the public firm, or a first price auction to be played by public and private firms symmetrically. The two step procedure is chosen when (6) is lower than (10) in expected terms, namely when:

\[ E[P_{PSB}] = (n + 1) \int_{\tilde{t}_0}^{d} \left( 1 - G(\tilde{t}_0) \right)^n g(\tilde{t}_0) \, d\tilde{t}_0 + \]

\[ + n \int_{\tilde{t}_0}^{d} \int_{\tilde{t}_0}^{d} (1 - G(s))^n g(\tilde{t}_0) \, d\tilde{t}_0 > \]

\[ E[P_{TS}] = \int_{\tilde{t}_0}^{d} \left( 1 - G(t_*) \right)^n g(\tilde{t}_0) \, d\tilde{t}_0 + \]

\[ + n \int_{\tilde{t}_0}^{d} \int_{\tilde{t}_0}^{d} \tilde{t} (1 - G(\tilde{t}))^{n-1} g(\tilde{t}) \, d\tilde{t} g(\tilde{t}_0) \, d\tilde{t}_0 + \]

\[ + n \int_{\tilde{t}_0}^{d} \left( \int_{\tilde{t}_0}^{d} (1 - G(\tilde{s}))^{n-1} g(\tilde{t}_0) \, d\tilde{t}_0 \right) \int_{\tilde{t}_0}^{d} \left( 1 - G(\tilde{t}) \right)^{n-1} g(\tilde{t}) \, d\tilde{t} \]

The latter inequality reduces to:

\[ \int_{\tilde{t}_0}^{d} (1 - G(\tilde{t})) \left[ (1 - G(\tilde{t}))^{n-1} (1 + (n - 1)G(\tilde{t})) - (1 - G(t_*) \right)] \, d\tilde{t} > 0 \]

We therefore conclude that in general the results depends upon the distribution of firms’ costs \(G(\cdot),\) and the number of private bidders \(n.\) Consider for example the case of firms’ costs uniformly distributed on the unit interval. In this case the inequality (13) can be solved to yield:

\[ \frac{1}{n + 1} + \frac{(n - 1)}{(n + 1)(n + 2)} > \frac{(2^{-n} + 2n)}{(n + 1)(n + 2)} \]

which can be represented by Figure 2.

The two-stage procedure always generates an expected procurement cost lower than the one obtained from a FPSB auction with the public firm. Given the results of Proposition 1 we also conclude that the expected procurement
cost from the two stage procedure is lower than the one obtained in one-stage SPSB and English auctions as well.

We can conclude that, whenever there exists a welfare maximizing public firm competing against \( n \) private firms for the awarding of a procurement, the Government can reduce his expected cost if he keeps the public firm outside the auction and arranges with her a private deal for the payment. Once the public firm cost is known, the Government sets a ceiling price in an auction where only private firms participate. Such singling out is profitable in expected terms both for the Government and for the public firm.

5 Concluding comments

Having a public firm operating in the same industry together with \( n \) private firms may help the Government to lower his expected payment when he must award a procurement contract through an auction. In the paper we considered some simple alternatives open to the regulator and we showed that the presence of a welfare maximizing public firm may modify some standard results of auction theory. Our conclusions can be summarized as follows. The lowest procurement cost can be obtained when the Government is able to design a procedure that strictly favors the public firm over the private ones. We analyzed a two-stage procedure in which the public firm reveals separately her procurement payment, whereas private firms compete among each other and against an optimally set ceiling price. In so doing, the public firms gets the contract with higher probability with respect to the case in which she would have participated into a standard auction.

When on the contrary it is forbidden any asymmetric treatment of bidders, and hence the public firm must participate to the auction, we showed that the
expected procurement cost depends upon the auction procedure selected by the Government. The first price sealed bid auction performs better than the second price auction. We also showed that the policy of revealing to bidders the presence of a welfare maximizing public firm by means of an open auction does not generate further cost savings for the Government.

References


