How to model bidding behavior in electricity markets

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Modelling Electricity Markets

- Electricity markets design is constrained by some peculiar characteristics of the good "electricity"

1st Demand and supply must be kept in balance minute by minute. For this reason a System Operator must be responsible for dispatch of generation

2nd The day-ahead market must be complemented by Intraday sessions and Real time markets to accommodate variations in demand/supply.

3rd The allocation rule is based upon a "cost minimizing" perspective (merit order of bids).

4th All dispatched units are paid the same equilibrium price (uniform price auction). Real time markets adopt the pay-as-bid payment rule.

Two modelling approaches

- A The auction approach is based on the private value model (production costs are the private information of bidders). Since the auction allocate multiple units, the appropriate auction model is known as the Share Auction model, first studied by Wilson (QJE, 1989).
- SFE The supply function equilibrium approach is based upon common knowledge of costs and demand uncertainty. Klemperer and Meyer (Econometrica, 1989) study the Nash equilibrium in supply functions that is committed to by agents BEFORE the realization of market demand.

- Both approaches have been generalized so as to take into account bilateral contracts and firm vertical integration.
- On the first perspective we have Green and Newbery (1992) and Newbery (1998), on the second perspective we have Wolak, Hortaşçu and Puller and, for the Italian market, Bosco, Parisio and Pelagatti.

The framework of Wilson

- Assume you want to buy a share of a total quantity Q in a competitive/pay-as-bid auction.
- Assume that each bidder $i, i \epsilon N$,has a marginal valuation function $v(q, s_i)$, decreasing wrt q
- Assume that s_i represents "private information" with $s_1, ..., s_N$ iid random variables drawn from distribution $F(.)$
- Equilibrium bidding strategies are functions $y(p, s_i)$: namely, bidder i is willing to buy quantity y at price p, given private information s_i.
- Call
$$RS_i = Q - \sum_{j \neq i} y(p, s_j) \tag{1}$$
the residual supply for bidder i in a symmetric equilibrium

The framework

- Market clearing occurs at price p_c when

$$\sum_{i=1}^{N} y_i(p_c) = Q \text{ or}$$
$$RS_i(p_c) = y(p_c, s_i)$$

- Define the probability distribution of the market clearing price as follows:

$$\begin{aligned} H\left(p, y_i(p)\right) &= Pr\left[y_i(p) \leq RS_i(p)\right] \\ &= Pr\left[p_c \leq p\right] \end{aligned}$$

$\left[\underline{p}, \overline{p}\right]$ is the price interval, or the support of the distribution.

The bidder's problem

- Maximize the expected profit as follows:

$$max_{y(.)} \int_{\underline{p}}^{\overline{p}} (\Pi_i(y_i(p), p)) dH\left(p, y_i(p)\right) \tag{2}$$

- Integrate (2) by parts (K is a constant of integration to be determined on the basis of initial conditions):

$$K - \int_{\underline{p}}^{\overline{p}} \frac{\partial}{\partial p} \Pi_i(y_i(p), p) H(y_i(p), p) dp \tag{3}$$

where $\frac{\partial}{\partial p} \Pi_i(y_i(p))$ is equal to:

$$[(v(y_i(p), p) - p)y'] \text{ in pay-as bid auctions}$$
$$\left(v_y(.)y' + v_p(.) - 1\right) y(p) - \left(v(y_i(p), p) - p\right) y' \text{ in u-price auctions}$$

Solutions

- The Euler-Lagrange necessary conditions for an optimal $y(.)$ yield:
- Pay-as-bid

$$p = v\left(y_i(p), p\right) - \frac{H(y_i(p), p)}{H_p}$$

- Uniform

$$p = v\left(y_i(p), p\right) + y_i(p)\frac{H_y}{H_p}$$

- In both cases the valuation is reduced by a (different) shading factor
- Truth telling is not optimal

More on the shading factor

- $H(p_c, y_i(p_c))$ is the distribution of the equilibrium price when bidder i asks y_i at price p_c
- $H_y(p_c, y_i(p_c))$ indicate the shift in the probability distribution of price due to a change in the quantity bought by bidder i. It can be roughly be interpreted as a measure of individual market power. Notice that this term is always non-positive since increasing demand increases the equilibrium price *ceteris paribus*. This term contributes to the shading factor for uniform-price auction only. The bidder bids his valuation when $H_y = 0$. We call this differential bid shading.
- $H_p(p, y_i(p))$ is the density of the market clearing price when bidder i asks y_i and the other bidders follow a symmetric bidding strategy.

How can we use the model in electricity auctions?

1st A bidder now is a seller. The valuation $v(y(p), p)$ is replaced by marginal production cost $C_i'(y(p))$

2nd Use the probability distribution of the equilibrium price to define the expected profit of a bidder as follows:

$$E\left[\Pi_i\right] = \int_p \left[py_i(p) - C\left(y_i(p)\right)\right] dH\left(p, y_i(p)\right)$$

3rd Derive the bidding function (uniform price)as:

$$p - C'(y_i(p)) = y_i(p)\frac{H_{y_i}\left(p, y_i(p)\right)}{H_p\left(p, y_i(p)\right)}$$

4th Interpretation of:

$$\frac{H_{y_i}\left(p, y_i(p)\right)}{H_p\left(p, y_i(p)\right)}$$

Uniform price auction for electricity

The ratio H_{y_i}/H_p can be interpreted as the shift in prob. dbn due to a change in the supply of i (see above) over the pdf associated with:

$$\left[p \leq p^c \mid y^*_{j \neq i}\right]$$

Consider that now H_y is non-negative since an increase in supply lowers the equilibrium price and hence it increases the probability that the equilibrium price is lower than p_c
How to derive the Numerator and Denominator of the above formula?

Making things simpler...

Hortacsu and Puller (2007) assume that: $y_i(p)$, i.e. the supply of bidder i (at a posted price p) is a continuously increasing differentiable function and use this to derive a manageable expression for the above ratio. Assume that: $y_i(p) = \alpha_i(p)$. Then:

$$p - C' = -\frac{y_i}{\frac{\partial}{\partial p} RD_i(p)} \tag{4}$$

where, in equilibrium, $RD'(p) = D'(p) + \frac{\partial}{\partial p} \sum_{\forall j \neq i} y_j(.)$ When the demand function is price-inelastic we have:

$$(p - C') = -\frac{y_i(p)}{\sum_{i \neq j} \alpha'(p)}$$

Making things simpler...

Rearranging (4), we obtain a measure of the ability of a firm to impact the market clearing price as follows:

$$\frac{(p - C')}{p} = -\frac{1}{\eta_i(p)} \tag{5}$$

where the rhs is the own-price elasticity of the residual demand curve faced by firm i evaluated at price p. This gives the percentage increase in the market clearing price that results from a one percent reduction in the firm's actual output.

Assumptions

- There is no uncertainty about production costs
- Firms are uncertain about demand
- Once demand is realized, the spot price is determined
- We do not consider capacity constraints and bilateral contracts
- In this situation firms commit to a price-quantity schedule and not to a specific pair.
- Define net demand in the two-firms case as:

$$S^i(p) = D(p, t) - S^j(p)$$

- The market manager finds the lowest price that equates demand and supply

The problem

Klemperer and Meyer (Econometrica, 1989) prove that price-output pairs can be represented by a supply function $q_i = S^i(p)$ which is the solution of the following profit maximization problem:

$$Max_p \pi_i(p) = p\left[D(p) - q_j(p)\right] - C\left[D(p) - q_j(p)\right] \qquad i \neq j \qquad (6)$$

F O C can be written as:

$$\frac{\partial q_j}{\partial p} = \frac{q}{p - C'(q)} + D_p$$

The solution represents a Nash equilibrium in supply functions.

Behavior of the differential equation

We consider points (q, p) such that the supply function is increasing and it has a well defined positive directional slope, namely $0 < dq/dp < \infty$:

$$C'(q) < p < C'(q) - \frac{q}{D_p}$$

It can be shown that, depending on the number of firms, there is a connected set of equilibria bounded by an upper and lower supply schedule. The upper schedule is also called the Cournot line (meaning that the quantity supplied by the other firm is taken as given):

$$p = C' + \frac{q}{-D_p} = \frac{p - C'}{p} = \frac{1}{\eta}$$

The lower schedule coincides with price equal to marginal cost (competitive solution). Uncertainty of demand with unbounded upper support, zero minimum demand and linear demand and marginal costs guarantee that the equilibrium is unique.

Bilateral contracts

The majority of auction markets around the world allows participants to secure part of their production selling bilateral contracts to buyers. The advantage is to have a planned schedule of production at a certain price.
In Italy, bilateral contracts are subject to the control of the AEEG and they are included into the merit order as supply at zero price.
It is easy to prove that the presence of bilateral contracts modifies the incentive for bidders to exploit their potential market power.
Contracts may be pro-competitive.

Auction approach

The max problem changes as follows:

$$E\left[\Pi_i\right] = \int_p \left[(py_i(p) - C\left(y_i(p)\right)) + (\bar{p} - p)x_i\right] dH\left(p, y_i(p)\right)$$

where $\bar{p}$ indicates the agreed contract price and x_i is the contracted quantity. This is the so-called contract for differences. The optimal bid now becomes:

$$p - C'(y_i(p)) = (y_i(p) - x_i)\,\frac{H_{y_i}\left(p, y_i(p)\right)}{H_p\left(p, y_i(p)\right)} \tag{7}$$

In (7) we see that the rhs is reduced and so the price/cost difference. What is relevant now is the NET position of the producer. It can be also a net buyer and in this case the bid will be lower than marginal production cost.